Monte Hall Problem

You're on a game show and presented three curtains. Behind these curtains there are two goats and one BRAND NEW CAR! You get whatever is behind the curtain you choose. Which curtain do you choose?

A B C

$P(A) = \frac{1}{3}$ $P(B) = \frac{1}{3}$ $P(C) = \frac{1}{3}$

Now, before the game show host reveals your prize, he reveals another curtain with a goat behind it.

A B C

Then he asks you if you'd like to switch your choice...?

Factorial Function (!)

Multiply all whole numbers from our chosen # down to 1

ex) $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

"Five Factorial"

n	n!		
1	**1**	1	1
2	2 × **1**	= 2 × **1!**	= 2
3	3 × **2 × 1**	= 3 × **2!**	= 6
4	4 × **3 × 2 × 1**	= 4 × **3!**	= 24
5	5 × **4 × 3 × 2 × 1**	= 5 × **4!**	= 120
6	etc	etc	

$$\boxed{n! = n \cdot (n-1) \cdots 1}$$

Q: Find 0!

$$n! = \frac{(n+1)!}{(n+1)}$$

$$4! = \frac{5!}{5} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5} = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

$$3! = \frac{4!}{4} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{4} = 6$$

$$2! = \frac{3!}{3} = \frac{3 \cdot 2 \cdot 1}{3} = 2$$

$$1! = \frac{2!}{2} = \frac{2 \cdot 1}{2} = 1$$

$$0! = \frac{1!}{1} = \frac{1}{1} = 1$$

★ $n!$ describes the # of ways we can Arrange n items

ex| How many ways can we arrange letters a, b, c ?

abc
acb
bac
bca
cab
cba

6 ways

$n=3$ because we are arranging 3 items (a, b, c)

$n! = 3! = 3 \cdot 2 \cdot 1 =$ 6 ways

ex| How many ways can you shuffle a deck of cards?

$52!$

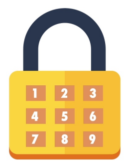
ex| Find $\frac{8!}{6!} = 8 \cdot 7 = 56$

ex| Find $\frac{9!}{5!4!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1} \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 126$

Permutation

Order **Does** Matter

"The combination to the safe is 472."
Now we do care about the order. "742" won't work, nor will "247." It has to be exactly "472"



Should be called a Permutation lock instead of a Combination lock

With Repetition

permutations can repeat

ex) In the lock above there are 9 numbers to chose from and we chose 3 of them

$$\underline{9}\ \underline{9}\ \underline{9} = 9^3 = 729 \text{ permutations}$$

$$\boxed{n^k}$$

n is the # of things to choose from and we choose k of them

ex) How many 5 digit whole numbers exist?

0, 1, 2, 3, ...

$$\underline{9} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} = 90000$$

Can't start with zero

Without Repetition

permutations cannot repeat

ex| What order could 16 pool balls be in?

We are unable to repeat any pool balls since we only have one of each

$\underline{16} \cdot \underline{15} \cdot \underline{14} \cdot \underline{13} \cdot \underline{12} \cdot \underline{11} \cdot \underline{10} \cdot \underline{9} \cdot \underline{8} \cdot \underline{7} \cdot \underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} =$ 21 Trillion

What if we just want to choose 3 pool balls?

$\underline{16} \cdot \underline{15} \cdot \underline{14} =$ 3,360 permutations

In other words, there are 3,360 different ways that 3 pool balls could be arranged out of 16 balls. Without repetition our choices get reduced each time.

$$P(n,k) = \frac{n!}{(n-k)!}$$

n is the # of things to choose from and we choose k of them

$$\frac{16!}{(16-3)!} = \frac{16!}{13!} = 16 \cdot 15 \cdot 14 = 3{,}360$$

ex| How many ways can 1st and 2nd place be awarded to 10 people?

$$P(10,2) = \frac{10!}{(10-2)!} = \frac{10!}{8!} = 10 \cdot 9 = 90 \text{ ways}$$

Combination

Order Does NOT Matter

"My fruit salad is a combination of apples, bananas, and grapes."
We don't care what order the fruits are in, they could also be "bananas, grapes, and apples" or "grapes, apples, bananas" and it will still be the same fruit salad.

ex) How many ways can you choose 2 people from a group of 4?

A B C D

Permutations	Combinations
AB BA CA DA	AB ~~BA~~ ~~CA~~ ~~DA~~
AC BC CB DB	AC BC ~~CB~~ ~~DB~~
AD BD CD DC	AD BD CD ~~DC~~
12	6
$P(4,2) = \frac{4!}{(4-2)!} = 12$	$C(4,2) = \frac{P(4,2)}{2!} = \frac{12}{2!} = 6$

"N choose K"

$$\boxed{C(n,k) = \frac{P(n,k)}{k!} = \binom{n}{k} = {}_nC_k = \frac{\#\text{ of Permutation}}{\text{Redundancies}}}$$

Going back to our pool ball example. We have 16 balls and we will choose 3, but this time order doesn't matter

Recall when order mattered (Permutations) we found

$\underline{16} \cdot \underline{15} \cdot \underline{14} = 3{,}360$ permutations

let's say balls 1,2,3 are chosen

Order does matter	Order doesn't matter
1 2 3 1 3 2 2 1 3 2 3 1 3 1 2 3 2 1	1 2 3
Permutation	Combination

$$\binom{16}{3} = \frac{16!}{3!(16-3)!} = \frac{16 \cdot 15 \cdot 14}{3 \cdot 2 \cdot 1} = 560 \text{ permutations}$$

Find the number of ways in which ten different books can be given to Ethan, Henry, Joshua, and Lucy if Ethan is to receive 4 books, Henry is to receive 3 books, Joshua is to receive 2 books and Lucy is to receive 1 book.

Ethan gets 4 books and then

Henry gets 3 books and then

Joshua gets 2 books and then

Lucy gets 1 book

$$\binom{10}{4}\binom{6}{3}\binom{3}{2}\binom{1}{1} = 12,600$$

A school basketball team of 5 students is selected from 8 boys and 4 girls.

(a) Determine how many possible teams can be chosen

$$\binom{12}{5}$$

(b) Determine how many teams can be formed consisting of 3 boys and 2 girls.

$$\binom{8}{3}\binom{4}{2}$$

(c) Determine how many teams can be formed consisting of at most 3 girls.

$$\binom{12}{5} - \binom{8}{1}\binom{4}{4}$$